On the expansion of the crack-tip craze during fatigue fracture in poly(methyl methacrylate)

J. Botsis
Department of Civil Engineering, Mechanics and Metallurgy, University of Illinois at Chicago, Box 4348, Chicago, IL 60680, USA
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The expansion of the crack-tip craze in poly(methyl methacrylate) during quasi-static fatigue fracture is considered. The driving force is taken in the form proposed by the Crack layer model. It is evaluated on the basis of experimentally measured quantities of the craze length and craze width and the assumption of a uniform stress applied along the boundary of the crack-tip craze (Dugdale–Barenblatt model). Correlation of the rate of expansion and the corresponding driving force suggests that a power-type kinetic equation adequately describes the expansion of the crack-tip craze under the investigated loading conditions.

(Keywords: damage; poly(methyl methacrylate); crack-tip craze; crack layer; expansion; expansional force)

INTRODUCTION

Mechanistic investigations of fatigue crack propagation (FCP) in a variety of engineering materials demonstrate that damage accumulation, within a zone adjacent to the crack tip, precedes slow crack growth. This zone is usually called a plastic zone[1], process zone[2,3], active zone[4], etc. It has been observed that for polymers damage constituting a process zone may be in the form of a single craze[5–8], multiple crazes[9–11], shear bands[12], a combination of both[13] or homogeneous deformation[14].

The specifics of such a fracture process is that damage activity within a process zone dissipates a substantial amount of energy which in mechanical experiments is manifested by the dependence of conventional toughness parameters on damage evolution and crack deceleration.

It therefore seems imperative that formulation of kinetic equations for crack-damage evolution would be the most desirable goal since damage plays an important role in the fracture behaviour of materials. This implies detailed information about the nature and kinetics of damage elements, their distribution and interaction, as well as their dependence on loading history. However, such a detailed characterization of damage seems unrealistic at present because of problems encountered in both experimental and theoretical studies.

Another approach to the problem, first introduced by Chudnovsky[4], is to treat the crack and its surrounding damage as a single macroscopic entity, namely, a crack layer (CL), and to employ principles of irreversible thermodynamics to model a fracture process. A schematic of a CL is shown in *Figure 1a*. The zone ahead of the crack tip where damage accumulates prior to crack growth is the active zone. The part of a CL complementary to the active zone is the inert zone.

According to the CL model fracture propagation results from active zone movements i.e. translation and rotation as a rigid body, self-similar expansion and distortion. The rates of these movements are treated as thermodynamic fluxes. The corresponding forces are introduced within the framework of irreversible thermodynamics[4].

In this report we attempt to describe the self-similar expansion of the crack-tip craze during rectilinear FCP in poly(methyl methacrylate) (PMMA). The force responsible for expansion is taken in the form proposed by the CL model[4]. Its evaluation is based on experimentally measured quantities of the craze length and width and the assumption of a uniform stress along the boundary of the crack-tip craze. Thereafter, a power-type and Arrhenius exponential relationships between the rate of expansion and the driving force are confronted with the experimental data on the expansion rate of the crack-tip craze.

EXPERIMENTAL

In situ experimental observations suggest that FCP in high-molecular-weight PMMA is preceded by a single craze[5–8]. A schematic of a crack-tip craze configuration is shown in *Figure 1b*.

The data on the rate of expansion of the crack-tip craze analysed in this study is that reported in ref. 15. Compact tension specimens of commercial-grade PMMA with dimensions $8\times8\times4$ mm are tested using a sinusoidal tensile load of frequency $\nu=2$ Hz, load ratio $R=0.1$ and temperature of 23°C.

An optical method is used to take interference optical micrographs of the crack-tip region during the course of the fatigue loading. Analysis of the interference fringes yields the shape and size of crack-tip craze at desired phases of load history. Details of the experimental set-up and the method of analysis can be found elsewhere[7,8,16].

Although it is possible to extract the shape of the craze by optical interferometry, the location of the craze tip and hence the craze length cannot be measured directly. It is obtained by extrapolating the experimental points

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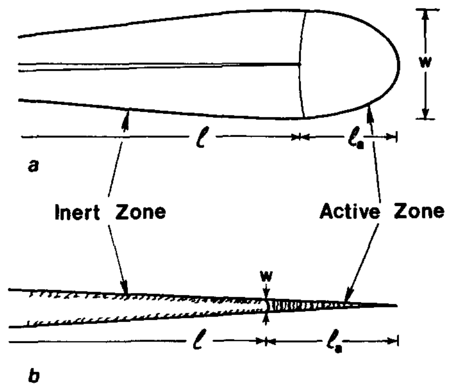

Figure 1 Schematics of (a) crack layer and (b) crack-tip craze

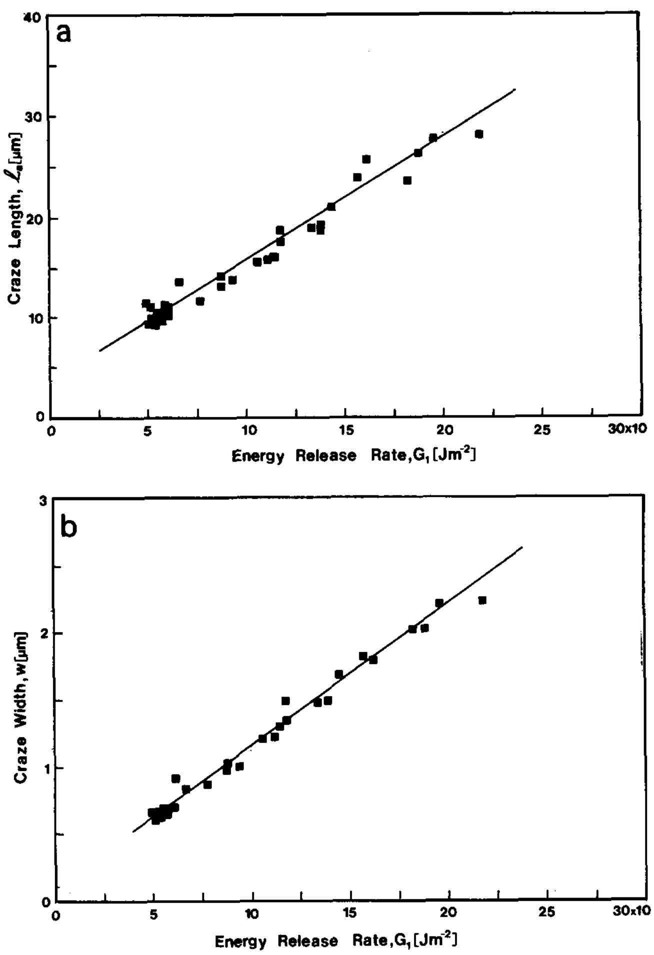

Figure 2 (a) Evolution of craze length l_a and (b) of craze width w with the energy release rate G_1

of the craze shape at a particular configuration and using a Dugdale–Barenblatt model[17,18].

For a compact tension specimen the energy release rate G_1 is calculated as:

$$G_1 = \frac{P^2(2W+l)^2}{t^2(W-l)^3E}F^2$$

where P is the applied load, W is the specimen width, l is the crack length, t is the specimen thickness and E is the Young's modulus of the material. The function F is a correction factor appropriate to the specimen geometry[19].

The evolution of the craze length l_a, craze width w (*Figure 1b*) and their ratio w/l_a as a function of G_1 are shown in *Figures 2* and *3*, respectively. The data clearly demonstrate that the crack-tip craze undergoes translation, expansion and distortion. Linear regression analyses of the data shown in *Figure 2* result in correlation coefficients of the order of 0.98. Thus, in the following, the dependence of w and l_a on G_1 is approximated by a straight line.

The rate of expansion is defined as $\dot{e} = \frac{1}{3}[\dot{V}(t)/V(0)]$, where $\dot{V}(t)$ is the rate of increase of craze volume at time t and $V(0)$ is its initial volume[4]. The evolution of $\dot{e}$ plotted against the energy release rate G_1 is shown in *Figure 4*. It is worth noting here that $\dot{e}$ changes by almost five orders of magnitude.

BACKGROUND

The response of a large class of engineering materials to a stress concentrator induced by a crack is the formation of a damage zone (or a so-called process zone) in a narrow region around the crack tip[5–14,20–24]. During fracture propagation a damage zone may experience translation, rotation and deformation[25].

Fracture mechanics approaches to FCP attempt to relate crack speed to some function of stress intensity factor K_1[26–28], or the energy release rate J_1[26–28]. This implies that fracture propagation can be described by one kinematic parameter, the crack length. Although

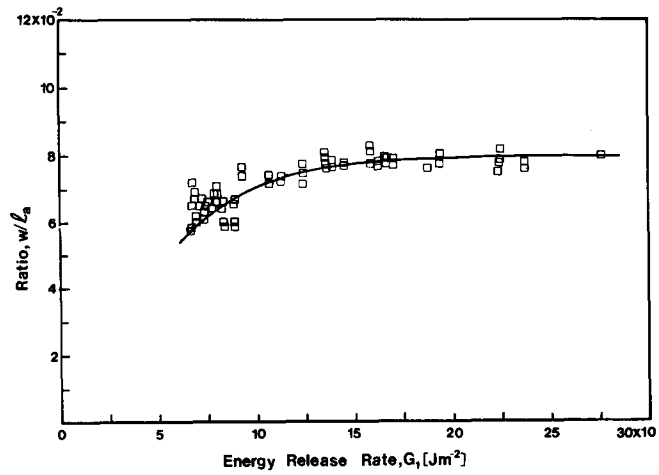

Figure 3 Variation of w/l_a with the energy release rate G_1

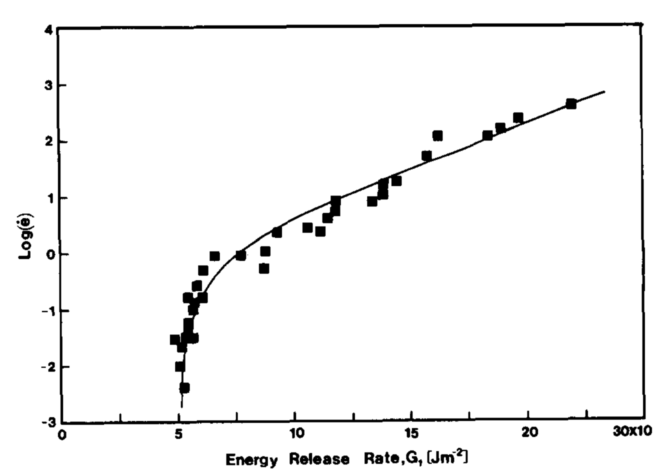

Figure 4 The rate of expansion $\dot{e}$ plotted against the energy release rate G_1

these models may well describe the particular data upon which they are formulated, they fail to describe crack deceleration observed in a number of materials[29–32]. Evidently, conventional fracture mechanics models of FCP characterize the translation of a damage zone only.

The phenomenological model of a CL describes a fracture process in terms of evolution of the damage zone[4]. In its scope CL is defined as the system consisting of the main crack and its surrounding damage (*Figure 1a*). The part of a CL ahead of the crack tip is the active zone[4,33]. Within this zone damage accumulates prior to crack advance. The part of a CL complementary to the active zone is the inert zone. In the case of PMMA examined in this paper, the active zone consists of the crack-tip craze (*Figure 1b*) and the inert zone of crazed material which remains on the fracture surface.

The CL model envisions the rates of the elementary movements of the active zone as thermodynamic fluxes. The forces corresponding to the fluxes are introduced using the principles of thermodynamics of irreversible processes[4,34].

Accordingly, the forces responsible for translation X^{tr} and self-similar expansion X^{exp} are:

$$X^{\mathrm{tr}} = J_1 - \gamma R_1 \qquad (1)$$

$$X^{\mathrm{exp}} = M - \gamma R_0 \qquad (2)$$

where J_1 and M denote the energies available for active-zone translation and expansion, γ is the specific energy of damage (a material constant)[35–37], and R_1 and R_0 are the resistance moments for active-zone translation and expansion and are expressed in terms of damage density and the geometry of the active zone[4]. The products of γ and R_1, R_0 represent the energies required for translation and expansion, respectively. Thus, relations (1) and (2) express the energy barriers for translation and expansion of an active zone.

As J_1 approaches γR_1 in (1), the crack speed tends to infinity and corresponds to the transition to fast fracture, at which:

$$J_{1c} = \gamma R_{1c} \qquad (3)$$

where J_{1c} and R_{1c} are the energy release rate and the resistance moments for CL translation at critical propagation[4].

For an elastic solid under plane loading conditions, the energy release rates J_1 and M in relations (1) and (2) are given by the following contour integrals[38,39]:

$$J_1 = \int_{C_1} (f n_1 - T_k u_{k,1})\, \mathrm{d}s \qquad (4)$$

$$M = \int_{C_2} (f x_i n_i - T_k u_{k,i} x_i)\, \mathrm{d}s \qquad (5)$$

where C_1 and C_2 are the contours of integration, f is the strain energy density, u_k is the displacement vector, T_k is the traction vector defined by the outward normal n_i to C_1 (C_2). It is important to mention here that the contour C_1 is taken around the crack tip while C_2 encloses the entire crack (*Figure 5a*)[39]. In addition J_1 and M are not independent. For a sharp crack of length $2l$ embedded into an infinite plate, M can be expressed as[40] $M = 2lJ_1$. The dependence of M on J_1 permits the evaluation of M (or J_1) when J_1 (or M) is known.

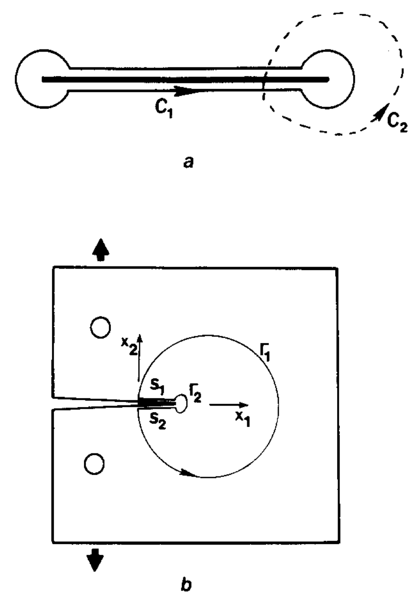

Figure 5 (a) Paths of integration for M and J_1 (see text for details). (b) Path of integration in a compact tension specimen

ANALYSIS AND DISCUSSION

Figure 5b presents the configuration of a compact tension specimen with the crack-tip craze. The J_1 and M integrals (relations (4) and (5)) may be evaluated by utilizing their path independence property. Thus, for the closed contour $\Gamma = \Gamma_1 + S_1 + \Gamma_2 + S_2$ we have:

$$J_\Gamma = J_{\Gamma_1} - (J_{S_1} + J_{\Gamma_2} + J_{S_2}) = 0$$

$$M_\Gamma = M_{\Gamma_1} - (M_{S_1} + M_{\Gamma_2} + M_{S_2}) = 0$$

Here J_{Γ_1} and M_{Γ_1} can be looked upon as the contributions of the external load. Furthermore, because of symmetry of specimen and the applied load the contributions along the contours S_1 and S_2 are equal. Thus:

$$J_{\Gamma_1} = 2J_{S_1} + J_{\Gamma_2} \qquad (6)$$

$$M_{\Gamma_1} = 2M_{S_1} + M_{\Gamma_2} \qquad (7)$$

Knowledge of the stress distribution along the contours S_1 (or S_2) and Γ_2 would permit evaluation of J_{Γ_1} and M_{Γ_1}. Attempts have been made to calculate the stress distribution across the boundary of the crack-tip craze zone[41–43]. These methods are based on experimentally measured displacements along the craze boundary.

However, it is argued that stress distributions obtained on the basis of the calculated displacements may be incorrect[44]. This is because the location of the craze tip and hence the displacements in the vicinity of the tip cannot be measured directly.

Nevertheless, the Dugdale–Barenblatt model[17,18] has been shown to provide a reasonable approximation of the crack-tip craze in PMMA[7,8]. Therefore, in order to evaluate the energy release rates (equations (6) and (7)) we employ the Dugdale–Barenblatt model to represent the distribution of stresses along the crack-tip craze boundary. According to this model, the stresses transmitted through the bulk material by the applied load and the stresses imposed by the crazed material produce singularities of the opposite sign at the crack tip. The singularities can be made to cancel each other by appropriate adjustment of the length of the nonlinear zone. This leads to $J_{\Gamma_2} = M_{\Gamma_2} = 0$. Furthermore, for small crack opening, $\oint n_1\,ds = x_i n_i = 0$. Thus, with the coordinate system at the crack tip equations (4) and (5) reduce to:

$$J_1 = 2\int_0^{l_a} \sigma_y(\partial u_2/\partial x_1)\,dx_1 \qquad (8)$$

$$M = 2\int_0^{l_a} -x_1\sigma_y(\partial u_2/\partial x_1)\,dx_1 \qquad (9)$$

where σ_y is the stress applied across the crack-tip craze boundary. In the following J_{Γ_1} and M_{Γ_1} are denoted by J_1 and M unless otherwise stated. Assuming that σ_y is constant across the boundary and approximating the craze by a triangle of height l_a and base w, integration of (8) and (9) yields:

$$J_1 = \sigma_y w \qquad (10)$$

$$M = \tfrac{1}{2}\sigma_y w l_a \qquad (11)$$

where w is the separation distance of the crack tip (*Figure 1b*). From (10) and (11) we obtain the following relationship between M and J_1:

$$M = \tfrac{1}{2} l_a J_1$$

which, for small extent of crazing (i.e. small-scale yielding), $J_1 = G_1 = K_1/E$, gives:

$$M = \tfrac{1}{2} l_a G_1 \qquad (12)$$

It is important to note at this point that the numerical coefficient in equation (11) depends upon the coordinate system employed. Physically, the coordinate system should be taken at the centre of expansion of the craze. Such data, however, are unavailable. In this analysis we take the centre of expansion to be at the crack tip.

Relation (12) expresses the active part of the expansional force (relation (2)). The resistive part γR_0 is approximated as follows[4]:

$$\gamma R_0 = \gamma\langle\rho\rangle A \qquad (13)$$

where $\langle\rho\rangle$ (g mm^{-3}) denotes the average density of the crazed material and A is the area of the crack-tip craze. The product $\gamma\langle\rho\rangle$ is calculated from the condition at critical propagation (equation (3)). Substituting for $R_{1c} = \langle\rho\rangle w_c$ (ref. 4), where w_c is the width of the crack-tip craze at critical propagation, we obtain:

$$\langle\rho\rangle\gamma = J_{1c}/w_c \qquad (14)$$

Furthermore, since the crack-tip craze is small compared to any other dimension of the specimen, small-scale yielding is assumed to prevail and $J_{1c} = G_{1c}$, where G_{1c} is the elastic critical energy release rate. Thus, from equations (12)–(14) we obtain the expansional force (relation (2)), i.e.:

$$X^{exp} = \tfrac{1}{2} l_a G_1 - (G_{1c}/w_c)A \qquad (15)$$

The evolution of X^{exp} with the energy release rate G_1 is shown in *Figure 6*.

The experimental data on the rate of expansion $\dot{e}$ (*Figure 4*) and the calculations of X^{exp} (*Figure 6*) are correlated next as an attempt to estimate their functional relationship. We restrict the analysis, however, to power and exponential relationships between $\dot{e}$ and X^{exp}, namely:

$$\dot{e} = B(X^{exp})^m \qquad (16)$$

$$\dot{e} = C_1 \exp(-C_2 X^{exp}) \qquad (17)$$

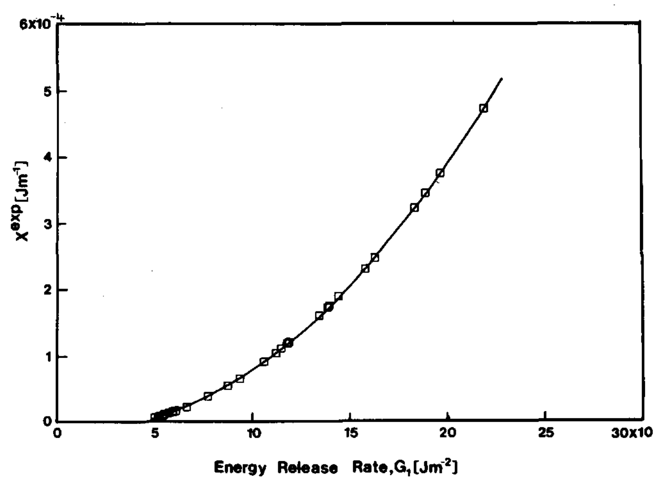

Figure 6 Expansional driving force as a function of G_1 (equation (15))

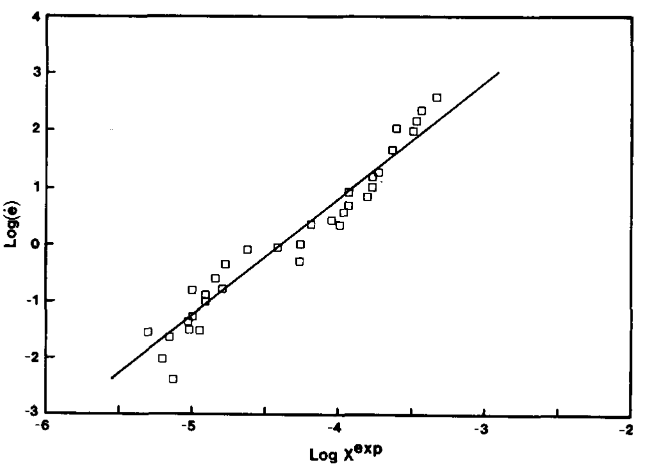

Figure 7 Variation of log($\dot{e}$) with log X^{exp} (equation (16))

The phenomenological parameters in (16) and (17) are obtained from linear regression analysis. Note that (16) reduces to the well known Onsager's linear relationship when $m=1$ and (17) is an Arrhenius-type exponential relationship with X^{exp} being the energy barrier for the process[45].

The straight line in *Figure 7* represents the right-hand side of (16). The data points are measurements of the rate of expansion (*Figure 4*). The correlation coefficient and the slope of the line are 0.96 and 2.1, respectively.

Craze growth is a rate-dependent irreversible process. The value of the slope, $m=2.1$, of the straight line in *Figure 7* suggests that a simple Onsager relationship cannot describe the expansion of the crack-tip craze. It further implies that, under the particular loading conditions, the expansion of the craze in PMMA does not fall under the linear range of irreversible thermodynamics or the so-called first-order thermodynamics[46,47].

Moreover, it is worth mentioning that, on the basis of dimensional analysis, Barenblatt *et al.*[48] suggest that power-type kinetic relations are the result of lack of characteristic time of the process under consideration. However, in order to verify this statement in the case of expansion of a crack-tip craze, additional experimental work under various loading conditions is required.

Application of relationship (17) should result in a straight line on the ln $\dot{e} - X^{exp}$ plane. The data shown in

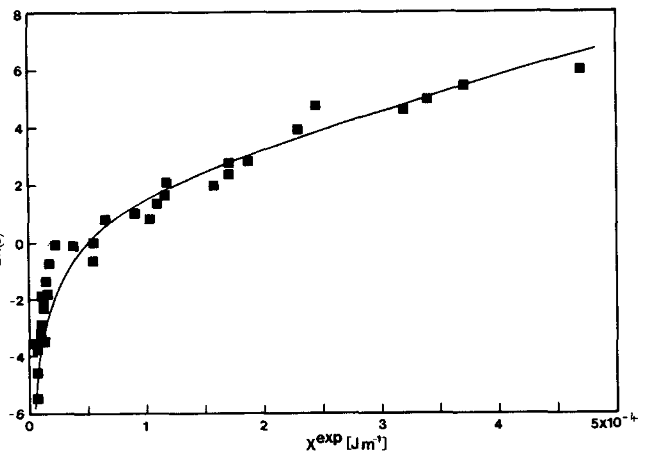

Figure 8 Variation of ln(ė) with X^{exp}

Figure 8, however, indicate that a single straight line cannot provide an adequate description of the range of the expansion rate analysed here.

CONCLUSIONS

The analysis presented above suggests that, for the particular loading history investigated, a simple power kinetic equation adequately describes the expansion of the crack-tip craze in PMMA. Additional experimental work, under various loading histories, is needed in order to prove its validity and lighten the nature of the phenomenological parameters.

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REFERENCES

1 Rice, J. R., in 'Fracture', (Ed. H. Liebowitz), Academic Press, New York, 1968, Vol. II, p. 191
2 Burech, F. E., in 'Fracture 1977', Pergamon Press, London, 1977, vol. 3, p. 929
3 Claussen, N. J. *J. Am. Ceram. Soc.* 1976, **59**, 49
4 Chudnovsky, A., 'Crack Layer Theory', NASA Contractors Report CR-17634, 1984
5 Kambour, R. P. *J. Polym. Sci. (A)* 1965, **3**, 1713
6 Bessenov, M. I. and Kurshinshii, E. V. *Sov. Phys.—Solid State* 1961, **3**, 1957
7 Brown, H. R. and Ward, I. M. *Polymer* 1983, **24**, 1213
8 Doll, W., Konczol, L. and Schinker, M. G. *Polymer* 1983, **24**, 1213
9 Bevis, M. and Hull, D. *J. Mater. Sci.* 1970, **5**, 983
10 Botsis, J., Chudnovsky, A. and Moet, A., 'Micromechanisms of Crack Layer Propagation in Polystyrene Under Cyclic Loads', XXIX ANTEC-SPE, Chicago, 1983, p. 444
11 Chudnovsky, A., Moet, A., Bankert, R. J. and Takemori, M. T. *J. Appl. Phys.* 1983, **54**, 5567
12 Takemori, M. T. and Kambour, R. P. *J. Mater. Sci.* 1981, **16**, 1110
13 Mills, N. J. and Walker, N. *J. Mater. Sci.* 1980, **15**, 1840
14 Andrews, A. H. and Barnes, S. P., 'Deformation, Yield and Fracture of Polymers', Plastics and Rubber Institute, Cambridge, 1982, 8.1
15 Konczol, L. and Sehanobish, K. *J. Macromol. Sci., Phys. (B)* 1987, **26**(3), 307
16 Schirrer, R., Schinker, M. G., Konczol, L. and Doll, W. *Colloid Polym. Sci.* 1981, **259**, 821
17 Dugdale, D. S. *J. Mech. Phys. Solids* 1960, **8**, 100
18 Barenblatt, G. I. in 'Advances in Applied Mechanics', Academic Press, New York, 1962, Vol. 67
19 Strawley, J. E. and Gross, B. *Eng. Fract. Mech.* 1972, **4**, 587
20 Donald, A. M. and Kramer, E. J. *J. Mater. Sci.* 1981, **16**, 2977
21 Evans, G. A. and Heuer, A. H. *J. Am. Ceram. Soc.* 1980, **63**, 248
22 Carpinteri, J. *J. Struct. Div., ASCE* 1982, **ST4**, 833
23 Haglan, G. R., Hahn, A. T. and Rosenfield, A. R. *Rock Mech.* 1973, **5**, 106
24 Chudnovsky, A. and Bessendorf, M., 'Crack Layer Morphology and Toughness Characterization in Steels', NASA Contractors Report 168154, 1983
25 Botsis, J. *Polymer* 1988, **29**, 457
26 Broek, D., 'Elementary Engineering Fracture Mechanics', Martinus Nijhoff, The Hague, 1982
27 Hellan, K., 'Introduction to Fracture Mechanics', McGraw-Hill, New York, 1984
28 Kanninen, M. F. and Popelar, C. H., 'Advanced Fracture Mechanics', Oxford University Press, New York, 1986
29 Andrews, H. E. and Walker, B. J. *Proc. R. Soc. (A)* 1971, **325**, 57
30 Bretz, P. E., Hertzberg, R. W. and Manson, J. A. *Polymer* 1981, **22**, 1272
31 Chudnovsky, A., Moet, A., Bankert, R. J. and Takemori, M. T. *J. Appl. Phys.* 1983, **54**, 5562
32 Sandt, A. and Hornbogen, E. *J. Mater. Sci.* 1981, **19**, 2915
33 Botsis, J., Chudnovsky, A. and Moet, A. *Int. J. Fract.* 1987, **33**, 263
34 Glansdorff, P. and Prigogine, I. in 'Thermodynamics Theory of Structure, Stability and Fluctuations', Wiley-Interscience, New York, 1971
35 Botsis, J., Chudnovsky, A. and Moet, A. *Int. J. Fract.* 1987, **33**, 277
36 Bakar, M. Ph.D. Dissertation, Case Western Reserve University, Cleveland, 1985
37 Nguyen, P. X. and Moet, A. *J. Vinyl Technol.*, 1985, **7**, 140
38 Rice, J. R. *J. Appl. Mech.* 1968, **90**, 379
39 Budiansky, B. and Rice, J. R. *J. Appl. Mech.* 1973, **40**, 201
40 Freund, L. B. *Int. J. Solids Struct.* 1978, **14**, 241
41 Kramer, E. J., in 'Crazing in Polymers', (Ed. H. H. Kausch), Springer-Verlag, Berlin, 1983, Vol. 52/53, 1
42 Brown, N. and Wang, Xue-qin *Polymer* 1988, **24**, 463
43 Koenczoel, L., Doell, W., and Beran, L., 'Deformation, Yield and Fracture of Polymers', Plastics and Rubber Institute, Cambridge, 1988, 17.1
44 Warren, W. E., Chudnovsky, A. and Mullen, R. L. *Polym. Eng. Sci.* in press
45 Moore, G. J., 'Physical Chemistry', Prentice-Hall, New York, 1974
46 Yourgrau, W., Van Der Merwe, A., and Raw, G., in 'Treatise on Irreversible and Statistical Thermophysics', Dover, New York, 1982
47 Nicolis, G. *Rep. Prog. Phys.* 1979, **42**, 225
48 Barenblatt, G. I. and Botivina, L. R. *Int. J. Eng. Sci.* 1982, **20**, 187